Vague chance?

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If there are events that are both vague and chancy, then those chances might not satisfy the axioms of probability. I provide an example of such vague chances, and demonstrate that whether or not "chance-probabilism" is true depends on your view on the logic of vagueness.

An urn contains seventy marbles in a range of hues. Ten marbles are blue, twenty green and forty are yellow. The marbles are well mixed, the drawing procedure is suitably fair, and the chance set up has all the other properties you might hope it to have. It is natural to say that the chance of drawing a blue marble is one seventh. It is also natural to say that it is more likely that you will draw a yellow marble than you will a green one. I would not consider someone who denied these judgments to be a competent user of the chance concept.

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Urn 1	Urn 2
10 Blue	10 Red
20 Green	20 Orange
40 Yellow	40 Red or Orange

Table 1. The two urns

Now consider a second urn which also contains seventy marbles in a range of hues; seventy distinct hues on a spectrum of red to orange. Ten marbles are determinately red and twenty are determinately orange. The remaining forty marbles are not determinately red and not determinately orange: they are borderline cases of red and of orange. What is the chance of drawing a red marble from this urn? A number that corresponds to the chance of drawing a red marble isn't as immediate in this case. Note that we can still say things like "it is at least as likely that you draw a red marble from urn 2 as it is that you draw a blue marble from urn 1". This suggests that we can say that the chance is at least one seventh: the determinately red marbles guarantee at least this much chance. And we can say that the chance of red is at most five sevenths: even if we included all the marbles that are such that it is vague whether they are red as well as the determinately red ones, we would only have five sevenths of the marbles in such a collection (because two sevenths are determinately orange, and thus determinately not red). We might say that drawing a non-green marble from urn 2 is at least as likely as drawing a red marble from urn 1. Not all such comparisons are so clear. Which is more likely: a red marble from urn 2 or a green marble from urn 1? One might be tempted to say there is no fact of the matter about which is more likely. But note that such a claim would involve denying a standard assumption about the nature of objective chances: that they conform to the calculus of probabilities. I call this claim *chance-probabilism*. One of the consequences of this chance-probabilism assumption is that events – things like "you draw a red marble from urn 2" – get assigned real numbers that reflect their chances. Thus, all chances are comparable, and the intuitive response to the vaguely coloured marbles example is blocked. That is, chance-probabilism cannot accommodate this intuitive desire to refrain from judging which of two events is more likely.

This paper is about this puzzle. My main conclusion is that the formal structure of chances needn't be probabilistic if the chancy events can be vague. What

structure they do have depends on your view on the logic of indeterminacy.

1. Chance and probability

Chance-probabilism is the claim that chances are probabilistic. Almost everyone who writes on chance makes this assumption, so much so that "probability" is often used as a synonym of chance.[1] For example early writers on the topic of probability often described their work as work on "chance": for example John Venn's *The Logic of Chance* and Thomas Bayes' *An Essay towards solving a Problem in the Doctrine of Chances*. One typically sees probabilities defined as functions over a Boolean algebra of events. We take a slightly different approach. The set of events or propositions that chances are defined over has a particular logical compositional structure. That is, if X is an event and Y is an event then $X \vee Y$ (X or Y) is an event and so is $X \wedge Y$ (X and Y). The set has privileged elements $\top$ and $\bot$ which stand for the necessary and the impossible event respectively. There's a notion of logical entailment $\vdash$ connected to this set. We can now define a probability as a real-valued function satisfying:

- If $\top \vdash X$ then $\mathbf{pr}(X) = 1$ and if $X \vdash \bot$ then $\mathbf{pr}(X) = 0$

- If $X \vdash Y$ then $\mathbf{pr}(X) \leq \mathbf{pr}(Y)$

- $\mathbf{pr}(X \vee Y) + \mathbf{pr}(X \wedge Y) = \mathbf{pr}(X) + \mathbf{pr}(Y)$

We don't explicitly mention negation, but if the structure has a unary connective that behaves as we expect negation to behave, then it follows that $\mathbf{pr}(\neg X) =$

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1. Two notable exceptions are Colyvan (2008) and Norton (2008, 2007). Though these authors are really focussed on reasoning about uncertainty and inductive inference respectively, they are among the few philosophers who can be understood as thinking about nonprobabilistic objective probabilities. Nonprobabilistic approaches to *credence* are more common, though still in a minority (Bradley, 2014).

$1-\mathbf{pr}(X)$. If ⊢ is classical logical entailment, then we get a Boolean algebra, as standard.[2] But this definition allows for us to define probabilities on other structures, for example Heyting algebras. These are to intuitionistic logic what Boolean algebras are to classical logic.[3] So there are two distinct chance-probabilism positions: one might demand that chances are *classical* probabilities, or one might allow that chances are probabilities with ⊢ interpreted in some non-classical way. Call these classical and revisionist chance-probabilism respectively. Williams (2012b,a, ming) explores revisionist probabilism for the case of subjective credences.

So chance-probabilism is the view that the set of events that have chances attached to them form some sort of algebraic structure, and that the chances are adequately represented by a probability function. Note that chances are represented by an *unconditional* probability: I have said nothing – and will say nothing – about conditionalisation. I mention this to emphasise that, despite prima facie similarities, I am engaged in quite a different project from Humphreys (1985). As Suárez (2013) notes, there are two sides to the thesis that identifies chances[4] with probabilities: there is the claim that all chances are probabilities (or are represented by probabilities), and there is the claim that all probabilities are chances (or can be interpreted as chances). Paul Humphreys gave two arguments, one addressing each of these claims.

The basic idea of Humphreys' first argument is that if $\mathbf{pr}(A|B)$ is a probabil-

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2. The quotient algebra of the symmetric part of ⊢ is a Boolean algebra.

3. We don't have space here to describe precisely what properties ⊢ must satisfy in order for this definition to make sense, but note, for example, that if the connectives are not commutative then the third condition might be unsatisfiable. See Bradley (ming) for more details.

4. The literature on Humphreys paradox often talks of "propensities" rather than "chances". I won't make a distinction between the two terms. Note that Suárez (2013, ming) uses the two terms to mean different things: propensities are dispositional properties of objects and chances are the manifestations of those properties.

ity that we are interpreting as the conditional chance of *A* given *B*, then in most circumstances $\mathbf{pr}(B|A)$ cannot be interpreted as a conditional chance. And the axioms of probability theory – via Bayes' theorem – tell us how to calculate the one from the other (and the unconditional probabilities of the events). This is so because chances have a kind of causal or temporal asymmetry that is not respected by probability theory.[5] Thus, not all probabilities are chances.

Humphreys then provides an example of a chance set-up where we are naturally drawn to assent to particular attributions of chances that are not consistent with the probability calculus. I won't discuss the argument here, but see Suárez (2013, ming); Lyon (2014) for discussion. Thus not all chances are probabilities.

So on some level, the literature stemming from Humphreys is pushing in the same direction that I am: the tight connection assumed between chances and probabilities is not warranted. But in more fine-grained terms, we are making importantly different claims. We both deny chance-probabilism. But chance-probabilism is really a conjunction of claims: chances are represented by a real valued function *and* that function is bounded *and* it is additive *and* conditional probabilities are related to unconditional probabilities through the formula $\mathbf{pr}(A \wedge B) = \mathbf{pr}(A|B)\mathbf{pr}(B)$. In denying this conjunction Humphreys and I are allies. But we are putting pressure on different conjuncts: Humphreys on the last, me on the first and third. Put another way, the final conjunct above – about conditional probabilities – plays a vital role in Humphreys' discussion, but no role at all in the present paper.

Suárez, responding to Humphreys' paradox, takes chances to be dispositions whose manifestations are (unconditional) probabilities; so despite his taking on board the lessons of Humphreys (1985), he still endorses the probabilism claim that I reject. That is, even among those who have learned Humphreys' lesson that probabilities are not propensities, there is still a widespread acceptance that

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5. See Milne (1986) for more along this line.

propensities are probabilities: in the sense that propensities are assumed to have an additive structure.

Before continuing, we should say something about why one should care about the formal structure of chances. I suggest two consequences denying chance-probabilism would have: chance–credence coordination and "probabilistic" causation. Let's consider Lewis' famous Principal Principle (Lewis, 1986). This says that your degrees of belief ought to conform to your knowledge of the objective chances. Without wanting to wade into the details of this tricky discussion, what is important for us is that PP entails: "If you know only that the chance of X is x then you ought to believe X to degree x". Now if chances are nonprobabilistic, then your credences ought to be so too. However, there are other norms that govern credence. One important one is, awkwardly, *credence-probabilism*: your degrees of belief ought to conform to the calculus of probabilities. So it seems that in order to keep the two norms from being in conflict with each other, chances need to be probabilistic. So, if you subscribe to credence-probabilism, and to PP, then you need to argue for chance-probabilism. That is, you need chance-probabilism to be true in order to rule out your norms being in conflict. More carefully: credence-probabilism, the Principal Principle and nonprobabilistic chance are not compatible. If you were committed to both norms, then that, and the fact that your all-things-considered norms cannot be in conflict would entail chance-probabilism. Lewis seems to take this approach. Lewis (1986) suggested that the Principal Principle "seems to…capture all we know about chance" (p. 86). Since he also took credence-probabilism for granted, he would presumably endorse some argument of the following form: "My credences are necessarily structured in a certain way, and my credences must track chances. Thus chances must be structured the same way". But this seems backwards. My beliefs should conform to how the world is, not the world to my beliefs in it. I shouldn't be able to learn about the structure of the world merely by reflecting on what structure

my beliefs ought to have.[6]

It seems to me that conflict with putative epistemic norms shouldn't be enough to adjudicate on the truth of a claim about the structure of the world. That is, conflict with putative norms like PP shouldn't be enough to guarantee the impossibility of nonprobabilistic chances. The incompatibility of the above three claims, plus the "vague marbles" example yield a good reason to deny one of PP or credence-probabilism.

In short, nonprobabilistic chance should prompt a rethink on the norms for belief. For example, how evidence about chances influences belief might be more subtle than originally thought. Or perhaps credence-probabilism should be jettisoned as a norm on belief. Either move would be a substantial departure from standard views on epistemology. One could thus see this paper as another reason to take seriously the recent barrage of interesting work on imprecise probabilities (see Bradley (2014) for an introduction). Indeed, vagueness as a reason to question *credence*-probabilism has already been discussed by, for example, Lyon (ming) and Williams (2012b,a, ming). One thing I shall not discuss in this paper, but which deserves more attention, is the question of how one ought to respond to vague evidence. Lyon discusses the merits of a "character-matching principle" – that vague evidence ought to prompt vague belief – as do Sturgeon (2008) and Wheeler (2014), and Fenton-Glynn (ms) asks specifically what sort of chance-credence coordination principle would be appropriate for "unsharp" chance information.

As another example of why the structure of chances is worth discussing, consider the project of giving a reductive account of causation in terms of "probability raising" (Hitchcock, 2011). Crudely put, the basic idea is that A causes B if A's being true raises the probability of B. What seems to really be at stake is

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6. I encourage someone with more patience for Kant to turn the above into a Transcendental argument for chance-probabilism.

not probability raising, but *chance raising*. So if some chances failed to be probabilities, this would have consequences for the scope of arguments made in this field. For example, *Bayesian networks* are an important tool for causal inference. A Bayesian network is a directed acyclic graph where there is a probability function over the nodes (which represent variables) and edges between nodes represent conditional dependencies between the nodes (See (Hitchcock, 2011, Section 3) or Pearl (2009)). An edge from node X to node Y represents the fact that the chance of Y changes conditional on what value X takes. If chances were sometimes nonprobabilistic, such a probabilistic representation might not be faithful to the facts. Perhaps some more flexible framework for causal reasoning would be more appropriate. For example, perhaps there should be a *set* of probability functions defined on the nodes of the graphs. Such a system is called a *credal network* (Cozman, 2000). Causal inference using such a model might lead to a more subtle and nuanced picture of causation, since, unlike in the case of a single probability measure, there are many distinct concepts of conditional dependence and independence for sets of probabilities (Cozman, 2012; de Cooman and Miranda, 2007).

2. Chance and indeterminacy

Recall that we have an urn that contains seventy marbles, ten red, twenty orange and forty indeterminate colours between red and orange (see Table 1 on p. 1). Let $\mathbf{ch}(X)$ be the function that returns the chance of event X. What can we say about this function given the description of the situation? A chance-probabilist confronted with this situation would have to say that $\mathbf{ch}(\mathrm{Red})$ takes some precise value. But which precise value? How much more than $\frac{1}{7}$ is $\mathbf{ch}(\mathrm{Red})$? No answer to this question seems justified. Such a view does not seem to do justice to the

vagueness of the situation.[7]

We also had some intuitive judgements that we would like our theory of vague chances to vindicate. We summarise these in the following list:

Red-Blue It is at least as likely that you draw a red marble from urn 2 as it is that you draw a blue marble from urn 1.

Red Bounds The chance of red is bounded below by $\frac{1}{7}$ and above by $\frac{5}{7}$.

No Fact There is no fact of the matter about whether drawing a red marble from urn 2 is more or less likely than drawing a green marble from urn 1.

My claim is not that these are undeniable intuitions that all right-thinking people have about chances in the chance set-up described above. They will, however, help us to distinguish the various positions I outline from each other. Only some views "get it right" about all of these.

What we are going to do now is go through several views about the truth values of sentences involving vague predicates and see what consequences they have for the structure of the chances of such sentences. This isn't meant to be an exhaustive survey: the goal is to demonstrate that the formal structure of chances can differ depending on your view of vagueness. We're going to look at several common views on vagueness including fuzzy logic, supervaluationism, truth value gaps, and epistemicism.

Let's start by looking at some views on vague chance stemming from fuzzy logic or "degree theory" approaches to vagueness. One might take inspiration from Smith (2010, 2008) in taking vague propositions to have "degrees of truth"

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7. Note that the argument here is stronger than the corresponding argument against credence-probabilism. In the credence case, the probabilist can just say that any particular values for red and orange are allowed: the value is not rationally constrained, but additivity is required. Here, since we are dealing with objective chances, the analogous move cannot be made: no determinate probabilistic answer to the question is permissible.

attached to them, and take chance to be expected truth value.[8] So each event gives rise to a function that maps each marble to the degree of truth of that proposition for that marble. So for the determinately red marbles, the function R outputs 1, and for the determinately orange (i.e. determinately not red) marbles, the function R outputs 0. Let's imagine that the marbles of vague colour have degree of truth $\frac{1}{2}$ for "Red" and for "Orange". That is, let's imagine that for those marbles, both the functions R and O output $\frac{1}{2}$. Further, let's imagine that there is a basic chance measure μ that assigns to each marble a chance of $\frac{1}{70}$: this reflects the fact that we take the marble-drawing process to be fair.[9] Now, the chance that a red marble is drawn is given by the expected truth value of the function R that represents that event. That is,

$$\mathbf{ch}_d(\text{Red}) = \sum_w \mu(w) R(w)$$

where the sum is taken over the marbles (or over the worlds that correspond to each marble's being drawn). Recall that there are ten marbles that are determinately red ($R(w) = 1$) and there are forty marbles such that it is vague whether they are red ($R(w) = \frac{1}{2}$). Thus, the chance of Red is $\frac{10 \times 1 + 40 \times \frac{1}{2}}{70} = \frac{3}{7}$. Doing the same calculation for Orange yields $\frac{4}{7}$. So far, so good: this looks like this determines a probabilistic chance function. That is, it looks like $\mathbf{ch}_d$ is additive. But what does Smith's theory tell us about the function $R \vee O$ that outputs the degree of truth of "Red or Orange"? He says that $(R \vee O)(w) = \max\{R(w), O(w)\}$ which is $\frac{1}{2}$ for the vague marbles.[10] This leads to a chance of "Red or Orange" of $\frac{5}{7}$. But all

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8. Smith is interested in degrees of belief, but the same moves can be made in the chance case.

9. Or better: there are seventy possible worlds, one for each marble's being drawn, and these possible worlds are equiprobable.

10. As Smith notes, there are, in fact, a number of rival kinds of truth function for disjunction. This one has some nice properties. Much of what I say applies, mutatis mutandis for other truth functions.

the marbles are red or orange![11] This might seem like it violates probabilism, but given that $(R \wedge O)(w) = \min\{R(w), O(w)\}$, so $\mathbf{ch}_d(\text{Red and Orange}) = \frac{2}{7}$, and thus additivity is still satisfied. But is a nonzero value for $\mathbf{ch}_d(R \wedge O)$ really sensible? Consider the following reasoning: it should be impossible for an object to exhibit incompatible properties; if something is impossible, then it has chance zero; but the degree theory assigns nonzero chance to an object exhibiting incompatible properties. So there is something wrong with the degree theory. Perhaps someone who was really committed to the "degree-theoretic" approach to vagueness would bite the bullet here and accept that there is a chance of drawing a marble that exhibits incompatible properties.

What the degree theory approach gets us is a revisionist chance-probabilism, where the relevant notion of entailment is: $X \vdash_{ND} Y$ iff for all w we have $X(w) \leq Y(w)$. This is what Williams (ming) calls "No Drop": no drop in truth value through entailment. This does not get us classical chance-probabilism. For example, imagine that at every world $X(w) = \frac{1}{2}$. Then $(\neg X)(w) = \frac{1}{2}$ as well.[12] Thus $(X \vee \neg X)(w) = \frac{1}{2}$. Since this is so for every world, $\mathbf{ch}_d(X \vee \neg X) = \frac{1}{2}$. But this doesn't invalidate revisionist chance-probabilism, since $X \vee \neg X$ is not a tautology according to this No Drop entailment relation. $\top(w) = 1$ for all w by definition, so we do not have that $\top \vdash_{ND} X \vee \neg X$.

Smith himself is explicit that he wants to marry his degree theory with a classical logic. He does this by defining a different degree-theoretic entailment relation that recaptures all the classical entailments. But such a view makes chances non-probabilistic, since (continuing the example from the last paragraph) $X \vee \neg X$ is a classical tautology but gets chance less than 1. So we have a "No Drop" degree

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11. (Smith, 2008, pp.85–7) points out that intuitions differ about whether "The marble is red or orange" is determinately true of a borderline case of red and orange.

12. On Smith's view, $\neg X(w) = 1 - X(w)$ for all w, though, again, there are alternative possible truth functions for negation.

theory that satisfies revisionist chance probabilism, or we have Smith's degree theory (with its classical logical entailment) that is nonprobabilistic in that it doesn't assign all tautologies – tautologies of classical logic – chance 1.

Degree theories of either flavour don't seem to do justice to the No Fact intuition. That is, there's always some particular number attached to the chance of each event, and such numbers can be compared, and thus there is always a fact of the matter about which of the two numbers is bigger (about which of the two events is more likely). In this case, $\mathbf{ch}_d(\text{Red}) = \frac{3}{7}$ while $\mathbf{ch}_d(\text{Green}) = \frac{2}{7}$. So there is a fact of the matter about which of Red or Green is more likely. The other two intuitions – Red Bounds and Red-Blue – are satisfied.

Let's turn now to "truth value gap" approaches. One thing we could do is simply deny that vague propositions have truth values, and deny that vague events have chances. So there is no chance that a marble drawn is red, only a chance that a marble drawn is determinately red. This position salvages chance-probabilism at the cost of doing violence to the intuitive view of what sort of things can have chances. In the case of credences, and betting on events, it is reasonable to require that the occurence or non-occurence of events gambled on can be unambiguously determined (Milne, 2008), but the analogous move for chance seems less warranted. Call this the "determinate events" view. Such a view accommodates No Fact, since if there's no chance of red, there's no fact about how that chance relates to other chances.[13] However, if there is no chance attached to "the next marble drawn will be red", then it is not the case that that chance is at least $\frac{1}{7}$, nor is it the case that it is more likely than drawing a blue marble from urn 1.

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13. There's an interesting parallel with probabilities in Quantum Mechanics, here. Chances in QM are probabilistic if the "events" one looks at are restricted to a commutative subalgebra of the Hilbert space (Wilce, 2012). But if you have a more liberal view of events, then the "chance" functions behave very strangely and nonprobabilistically (Hartmann and Suppes, 2010; Suppes and Zanotti, 1991).

So this view doesn't seem to do justice to the Red-Blue or Red Bounds intuitions.

How about we accept that chances don't straightforwardly attach to vague events, but then find a way to attach them derivatively? Consider taking "the chance of X" to mean the chance attached to the biggest (determinate) event smaller than X; that is, the event with the biggest chance that entails X. If we think of determinate events as measurable, and indeterminate events as unmeasurable, then this is the "inner measure". This function presumably coincides with the function that equates the chance of red with the chance of determinately red. Such a function would be superadditive but not additive, since $\mathbf{ch}_{in}(\text{Red}) = \frac{1}{7}$ and $\mathbf{ch}_{in}(\text{Orange}) = \frac{2}{7}$, but $\mathbf{ch}_{in}(\text{Red or Orange}) = 1$ since all marbles are either red or orange. Such an approach mirrors the approach that (Field, 2000, Section 5) takes in the case of credence. Call this the "inner measure" view.

This view might appear to not sanction the "No Fact" intuition, but it can be made to do so by reinterpreting what it means to say that X is more likely than Y. One might think that X is more likely than Y iff $\mathbf{ch}_{in}(X) \geq \mathbf{ch}_{in}(Y)$. But if instead we interpret this as: "X is more likely than Y iff $\mathbf{ch}_{in}(X) \geq 1 - \mathbf{ch}_{in}(\neg Y)$", then no relation of "more likely than" holds between the events of drawing a red marble and drawing a green marble. This may seem a strange move, but the "dual" function $\mathbf{ch}_{out}(X) = 1 - \mathbf{ch}_{in}(\neg X)$ can naturally be interpreted as the chance of being not determinately not red. If $\mathbf{ch}_{in}$ is a sort of "lower bound" on the chance of red, then the dual is a natural upper bound. If $\mathbf{ch}_{in}$ is a sort of inner measure, then the dual is the natural outer measure. The outer measure assigns the chance of red the value of the determinate event that entails "Red" that has the smallest chance. Note that $\mathbf{ch}_{out}(\text{Red}) = \frac{5}{7}$, which accords with the "Red Bounds" intuition. This sort of dual function will be familiar to those who know the theory of lower and upper probabilities.[14] We can think of $\mathbf{ch}_{in}$ and

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14. The "dual function" move doesn't help Smith accommodate the No Fact intuition, since in Smith's framework it's easy to show that $1 - \mathbf{ch}_d(\neg X) = \mathbf{ch}_d(X)$ for all X. All that is required

$\mathbf{ch}_{out}$ as defining an interval within which the chance would lie if the event were determinate. With this "interval" understanding of what's going on, we can reinterpret "*X* is more likely than *Y*" as "the lower end of *X*'s interval is above the upper end of *Y*'s interval". This relation is sometimes called *interval dominance*. So then we have $\mathbf{ch}_{int}(X) = [\mathbf{ch}_{in}(X), \mathbf{ch}_{out}(X)]$, an interval-valued function, and the relation of "more likely" is interpreted as $\mathbf{ch}_{in}(X) \geq \mathbf{ch}_{out}(Y)$. In the example $\mathbf{ch}_{int}(\text{Red}) = \left[\frac{1}{7}, \frac{5}{7}\right]$, $\mathbf{ch}_{int}(\text{Orange}) = \left[\frac{2}{7}, \frac{6}{7}\right]$, and $\mathbf{ch}_{int}(\text{Red or Orange}) = [1, 1]$. The chance intervals for Red and for Orange overlap, and thus neither is more likely than the other, using our modified interpretation of "is more likely than".

Let's move on to a supervaluationist approach now. One might say that it is vague what value "the chance of red" takes, but that that value is certainly somewhere between $\frac{1}{7}$ and $\frac{5}{7}$. Or one might think of all the "precisifications" of the example that determine a particular colour – red or orange – for each indeterminate marble and thus a particular (probabilistic) chance of drawing a red marble. One collects the set of probability functions determined in this way and call this the chance. More carefully, every completion of the gappy truth value assignment gives a (classical probabilistic) chance to each event. The set of these assignments can form the basis of an analysis of vague chance. We can either construct a set-valued function that outputs the set of chances of the completions for a given input, or we can take the set of chance functions to be the representing object. The interval valued function outputs the same intervals as the inner measure view would.[15] In either case, we clearly don't have chance-probabilism, since in either case, there is not a real-valued function that represents the chances.

We've seen how the set-valued function behaves when we met it when discussing the inner measure view,[16] so let's look more carefully at the set of func-

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is that $\neg S(w) = 1 - S(w)$ and that $\sum \mu(w) = 1$, both things Smith endorses.

15. This is theorem 2.3.3 of Halpern (2003).

16. The sets of values that this supervaluationist view produces needn't be intervals – i.e.

tions view. What does it mean to say that X is more likely than Y on such a view? If we treat X is more likely than Y as "determinately, X is more likely than Y" then there is no fact of the matter as to whether a red marble from urn 2 is more or less likely than a green marble from urn 1. This is so since some precisifications (completions of the gappy truth assignment) make green more likely, and some red. On all completions of the gappy truth value function, it is true that at most five sevenths of the marbles are red, and at least one seventh are red, so Red Bounds is satisfied, as is Red-Blue.

A final view of vagueness that we should look at is epistemicism (Williamson, 1994): the view that all vagueness is epistemic, all vagueness is just ignorance of the proper extension of the predicates we use. On this view, there is some particular partition of the marbles into red and orange that matches the actual – but unknown – extension of the predicates "Red" and "Orange". Some particular completion of the gappy truth value is the correct completion. This truth value function determines classically probabilistic chances for the events. The No Fact intuition is not satisfied, since there is a fact of the matter about which event is more likely. However, the spirit of the No Fact intuition is preserved in the fact that we cannot *know* which event is more likely. That is, while No Fact is false on this view, the following is true: "We cannot know whether drawing a red marble from urn 2 is more or less likely than drawing a green marble from urn 1". The other intuitions are satisfied, since whatever particular precisification is the correct precisification, it makes Red at least as likely as Blue, and will give Red a chance within the appropriate bounds.

Let's summarise the views on vagueness and their consequences for chance-probabilism

No Drop degree theory Revisionist chance-probabilism. Violates No Fact.

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convex – but that's not something that makes a difference for our discussion here.

Smith's degree theory Nonprobabilistic: tautologies needn't be assigned chance 1. Violates No Fact.

Determinate events Chance-probabilism holds, but at the cost of an unorthodox account of what the events are. Violates Red-Blue and Red Bounds.

Inner measure Chance functions are superadditive, but not additive. Satisfies the intuitions (given a particular interpretation of what it is for one event to be more likely than another).

Set-valued/Supervaluationist Chances are not described by real-valued functions, but by set-valued functions or sets of functions. Satisfies the intuitions.

Epistemicism There is a correct, but unknown, probabilistic chance function. Violates No Fact (but satisfies a nearby intuition).

3. Chance and statistics

Frequencies are probabilistic, and frequencies are evidence of chances. Hájek (1997) argues that it should at least be possible for chances to be described by frequencies, and thus that chances should at least be probabilistic. Paris (1994) offers an argument that is similar to Hájek's in that it shows how anything that is measured appropriately by statistics should be probabilistic. Note that these are *not* frequentist arguments. Whatever your attitude to chance, it seems that chances have some relation to frequencies. So even fans of propensity theories can take evidence from statistics as evidence for the structure of chancy powers.

There is a problem, however. Hájek's and Paris' arguments relied on the determinacy of the events. If it can be vague whether X and vague whether Y, but determinate that $X \vee Y$, then the statistics will inherit this vagueness and probabilistic representation will not be guaranteed unless you have particular views about

the logic of vagueness. Consider the statistics of the vague marbles example discussed earlier, where we stipulated that the marbles were all determinately Red or Orange. Let's imagine that you draw (with replacement) a large sample from the urn. Some of the time you will draw the marbles of indeterminate, borderline colour. How do you count them? They are unarguably red or orange, and thus should count towards the statistics of that disjunctive category. But should a marble that is not determinately red (but not determinately not red) count towards the statistics of red marbles? If you decided that it should not, then the statistics you would generate would be superadditive, but not additive. That is, the frequency of red or orange marbles would be strictly greater than the frequency of red marbles plus the frequency of orange marbles. This recalls the inner measure view discussed above. If instead we decided to count vaguely red marbles as "half a marble", then we'd get a sort of statistics that accords with a degree theory.

There is a research program in statistics that explores inference based on "chaotic probabilities" that are better accommodated by nonprobabilistic models (e.g. credal sets, lower previsions) than by standard models (Fine, 1988). This is further evidence that someone committed to chance-probabilism would struggle to make an argument for their position based on statistics.

4. Conclusion

When some event is both vague and chancy, there are a number of ways to cash out what the chance of that event is. Chances might be superadditive but not additive, or set-valued, or represented by sets of functions; which of these appeals to you depends on what view on the logic of indeterminacy you like. If you wanted to maintain chance-probabilism you could be an epistemicist, or you could adopt a degree theoretic "No Drop" revisionist chance-probabilism. So, those who wish to maintain – as many appear to have assumed by default – that chances are ad-

equately described by the probability calculus can only adopt one of a restricted set of views on the logic of indeterminacy. Even those who are happy to jettison chance-probabilism may find that which intuitions about chances of vague events they endorse will constrain which attitudes towards vagueness they can adopt. In short, vague chances are a useful diagnostic tool for vagueness.

5. Thanks

References

Bradley, S. (2014). Imprecise probabilities. In Zalta, E. N., editor, *The Stanford Encyclopedia of Philosophy*.

Bradley, S. (forthcoming). Nonclassical probability and convex hulls. *Erkenntnis*.

Colyvan, M. (2008). Is probability the only coherent approach to uncertainty? *Risk Analysis*, 28:645–652.

Cozman, F. (2000). Credal networks. *Artificial Intelligence*, 120:199–233.

Cozman, F. (2012). Sets of probability distributions, independence and convexity. *Synthese*, 186:577–600.

de Cooman, G. and Miranda, E. (2007). Symmetry of models versus models of symmetry. In Harper, W. and Wheeler, G., editors, *Probability and Inference: Essays in Honor of Henry E. Kyburg Jnr.*, pages 67–149. Kings College Publications.

Fenton-Glynn, L. (m.s.). Unsharp best system chances.

Field, H. (2000). Indeterminacy, degree of belief, and excluded middle. *Noûs*, 34:1–30.

Fine, T. L. (1988). Lower probability models for uncertainty and nondeterministic processes. *Journal of Statistical Planning and Inference*, 20:389–411.

Hájek, A. (1997). Mises redux – redux: Fifteen arguments against finite frequentism. *Erkenntnis*, 45:209–227.

Halpern, J. Y. (2003). *Reasoning about Uncertainty*. MIT press.

Hartmann, S. and Suppes, P. (2010). Entanglement, upper probabilities and decoherence in quantum mechanics. In Suárez, M., Dorato, M., and Rédei, M., editors, *EPSA Philosophical Issues in the Sciences: Launch of the European Philosophy of Science Association*, pages 93–103. Springer.

Hitchcock, C. (2011). Probabilistic causation. In Zalta, E. N., editor, *The Stanford Encyclopedia of Philosophy*. Winter 2011 edition.

Humphreys, P. W. (1985). Why propensities cannot be probabilities. *The Philosophical Review*, 94:557–570.

Lewis, D. (1986). A subjectivist's guide to objective chance (and postscript). In *Philosophical Papers II*, pages 83–132. Oxford University Press.

Lyon, A. (2014). From Kolmogorov to Popper to Rényi: There's no escaping Humphrey's paradox (when generalized). In Handfield, T. and Wilson, A., editors, *Chance and Temporal Asymmetry*, pages 112–125. Oxford University Press.

Lyon, A. (forthcoming). Vague credence. *Synthese*.

Milne, P. (1986). Can there be a realist single-case interpretation of probability? *Erkenntnis*, 25:129–132.

Milne, P. (2008). Bets and boundaries: Assigning probabilities to imprecisely specified events. *Studia Logica*, 90:425–453.

Norton, J. (2007). Probability disassembled. *British Journal for the Philosophy of Science*, 58:141–171.

Norton, J. (2008). Ignorance and indifference. *Philosophy of Science*, 75:45–68.

Paris, J. (1994). *The uncertain reasoner's companion*. Cambridge University Press.

Pearl, J. (2009). *Causality: Models*, *Reasoning and Inference*. Cambridge University Press, 2nd edition.

Smith, N. J. (2008). *Vagueness and Degrees of Truth*. Oxford University Press.

Smith, N. J. (2010). Degree of belief is expected truth value. In Dietz, R. and Moruzzi, S., editors, *Cuts and Clouds: Essays on the Nature of Logic and Vagueness*, pages 491–506. Oxford University Press.

Sturgeon, S. (2008). Reason and the grain of belief. *Noûs*, 42:139–165.

Suárez, M. (2013). Propensities and pragmatism. *Journal of Philosophy*, CX:61–92.

Suárez, M. (forthcoming). The chances of propensities. *British Journal for the Philosophy of Science*.

Suppes, P. and Zanotti, M. (1991). Existence of hidden variables having only upper probability. *Foundations of Physics*, 21:1479–1499.

Wheeler, G. (2014). Character matching and the Locke pocket of belief. In Lihoreau, F. and Rebuschi, M., editors, *Epistemology*, *Context and Formalism*, pages 185–194. Synthese Library.

Wilce, A. (2012). Quantum logic and probability theory. In Zalta, E. N., editor, *The Stanford Encyclopedia of Philosophy*.

Williams, J. R. G. (2012a). Generalised probabilism: Dutch books and accuracy domination. *Journal of Philosophical Logic*.

Williams, J. R. G. (2012b). Gradational accuracy and non-classical semantics. *Review of Symbolic Logic*.

Williams, J. R. G. (forthcoming). Non-classical logic and probability. In Hájek, A. and Hitchcock, C., editors, *Oxford Companion to Philosophy of Probability*. Oxford University Press.

Williamson, T. (1994). *Vagueness*. Routledge.